Countermeasures against Physical Attacks using Error-Correcting Codes

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Presentation Outline

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Definition

- Let k and n be two integers, such that $k \leq n$.
- The set of n-bit vectors, noted $\mathbb{F}_2^n$, is endowed with a structure of space vector.
- Let $\mathcal{C}$ be a subspace of $\mathbb{F}_2^n$ of dimension k.
- Then, $\mathcal{C}$ is a linear code of length n and dimension k.

Definition (supplement of a space vector)

$\mathcal{C}$ can be completed with some vectors in order to spawn $\mathbb{F}_2^n$. Those vectors define the supplement $\mathcal{D}$ of $\mathcal{C}$ in $\mathbb{F}_2^n$. We write $\mathbb{F}_2^n = \mathcal{C} \oplus \mathcal{D}$ to say that $\mathbb{F}_2^n$ is the direct sum of $\mathcal{C}$ and $\mathcal{D}$.

A linear code is spawned by a basis: the matrix whose rows consist in the basis vectors is called a *generating matrix*. We denote by G (resp. H) the generating matrix of $\mathcal{C}$ (resp. $\mathcal{D}$, the supplement of $\mathcal{C}$). Then, we have that every element $z \in \mathbb{F}_2^n$ can be written uniquely as:

$$\boxed{z = c \oplus d} \ , \tag{1}$$

where $c \in \mathcal{C}$ and $d \in \mathcal{D}$. Now, as all $c \in \mathcal{C}$ (resp. $d \in \mathcal{D}$) can also be written uniquely as xG (resp. yH), for a given $x \in \mathbb{F}_2^k$ (resp. $y \in \mathbb{F}_2^{n-k}$), we have the following equation:

$$\boxed{z = xG \oplus yH} \ . \tag{2}$$

Definition (minimal distance)

The minimal distance $d_{\mathcal{C}}$ of a linear code $\mathcal{C}$ of length n and dimension k is the minimal Hamming distance of any two different elements of $\mathcal{C}$. We say that $\mathcal{C}$ has parameters $[n, k, d_{\mathcal{C}}]$.

Definition (dual distance)

The dual distance $d_{\mathcal{C}}^{\perp}$ of a code $\mathcal{C}$ is the minimal Hamming weight $\mathrm{HW}(z)$ of a nonzero vector $z \in \mathbb{F}_2^n$ such as $\sum_{c \in \mathcal{C}} (-1)^{z \cdot c} \neq 0$, where $z \cdot c$ is the scalar product between z and c:

- $z \cdot c = \sum_{i=1}^{n} z_i c_i$, or equivalently
- $z \cdot c = z c^{\mathsf{T}} \in \mathbb{F}_2$ using matrix notations.

$n = 8$, $|\mathcal{C}| = 2^4$: Boolean linear code $[8, 4, 4]$

Dual distance $= 4$:

0x00	0	0	0	0	0	0	0	0
0x0f	0	0	0	0	1	1	1	1
0x36	0	0	1	1	0	1	1	0
0x39	0	0	1	1	1	0	0	1
0x53	0	1	0	1	0	0	1	1
0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
0xff	1	1	1	1	1	1	1	1

$n = 8$, $|\mathcal{C}| = 2^4$: Boolean linear code $[8, 4, 4]$

Dual distance > 1:

0x00	0	0	0	0	0	0	0	0
0x0f	0	0	0	0	1	1	1	1
0x36	0	0	1	1	0	1	1	0
0x39	0	0	1	1	1	0	0	1
0x53	0	1	0	1	0	0	1	1
0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
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0xf0	1	1	1	1	0	0	0	0
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0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
0xff	1	1	1	1	1	1	1	1

$n = 8$, $|\mathcal{C}| = 2^4$: Boolean linear code $[8, 4, 4]$

Dual distance > 2:

0x00	0	0	0	0	0	0	0	0
0x0f	0	0	0	0	1	1	1	1
0x36	0	0	1	1	0	1	1	0
0x39	0	0	1	1	1	0	0	1
0x53	0	1	0	1	0	0	1	1
0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
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0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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0x65	0	1	1	0	0	1	0	1
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0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
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0x9a	1	0	0	1	1	0	1	0
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0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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Dual distance > 3:

0x00	0	0	0	0	0	0	0	0
0x0f	0	0	0	0	1	1	1	1
0x36	0	0	1	1	0	1	1	0
0x39	0	0	1	1	1	0	0	1
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0x9a	1	0	0	1	1	0	1	0
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0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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0xf0	1	1	1	1	0	0	0	0
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0x39	0	0	1	1	1	0	0	1
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0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
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0x9a	1	0	0	1	1	0	1	0
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0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
0xff	1	1	1	1	1	1	1	1

Definition (orthogonal)

The orthogonal of a set $\mathcal{C} \subseteq \mathbb{F}_2^n$ is the space vector $\mathcal{C}^\perp$ defined as $\{d \in \mathbb{F}_2^n | \forall c \in \mathcal{C}, d \cdot c = 0\}$. When $\mathcal{C}$ is a linear code, $\mathcal{C}^\perp$ is called the dual code of $\mathcal{C}$. The generating matrix of $\mathcal{C}^\perp$ is called the parity matrix of $\mathcal{C}$.

Proposition

For a linear code $\mathcal{C}$, $d_{\mathcal{C}}^\perp = d_{\mathcal{C}^\perp}$.

Non-linear codes

- Systematic codes can be better than linear codes
- Ex. The Nordstrom-Robinson code $(16, 2^8, 6)$, though the BKLC is $[16, 8, 5]$.
- But the NR is ... $\mathbb{Z}_4$-linear ($[8, 4, 6]_{\mathbb{Z}_4}$, with $\{0, 1, 2, 3\} \rightarrow \{00, 01, 11, 10\}$).
- Distance polynomial:
  - $D_{\mathcal{C}}(X, Y) = \frac{1}{\text{Card}[C]} \sum_{x,y \in \mathcal{C}} X^{n-\text{HD}(x,y)} Y^{\text{HD}(x,y)}$.
  - $D_C(X, Y) = \sum_{d=0}^{n} B_d X^{n-d} Y^d$, where B_d is the normalized distance distribution, equal to: $\frac{1}{\text{Card}[C]} \text{Card}[\{(x, y) \in C \times C \text{ s.t. } \text{HD}(x, y) = d\}]$.
- The dual distance distribution $B_d^{\perp} \in \mathbb{Q}^+$ is the MacWilliams transform of the distance distribution, in the sense that $D_C^{\perp}(X, Y) = \frac{1}{\text{Card}[C]} D_C(X + Y, X - Y) = \sum_{d=0}^{n} B_d^{\perp} X^{n-d} Y^d$. The dual distance $d_C^{\perp}$ of C is the smallest $d > 0$ s.t. $B_d^{\perp} \neq 0$.

Constructions

- Cyclic codes, as an ideal of $\mathbb{F}[X]/(X^n - 1)$.
- Secondary constructions:
  - Puncturing, shortening, $(u, u + v)$, etc.
  - Example: [CG14a]. Let $\mathcal{C}$ be the QR (cyclic) $[17, 9, 5]$-code whose zeroes are β^i, $i = 1, 2, 4, 8, 9, 13, 15, 16$ where β is a primitive n-th root of unity. The generator polynomial of $\mathcal{C}$ is $X^8 + X^7 + X^6 + X^4 + X^2 + X + 1$. The shortened $\mathcal{C}_{\{17\}}$ has parameters $[16, 8, 5]$.
- BCH bound: longest string + 1; QR codes: $d_{\mathcal{C}} \geq \sqrt{n}$.

```
C:=QRCode(FiniteField(2),89);
MinimumDistance(C)
# ==> 17
```

but $BCH = 3, \lceil \sqrt{89} \rceil = 10$.

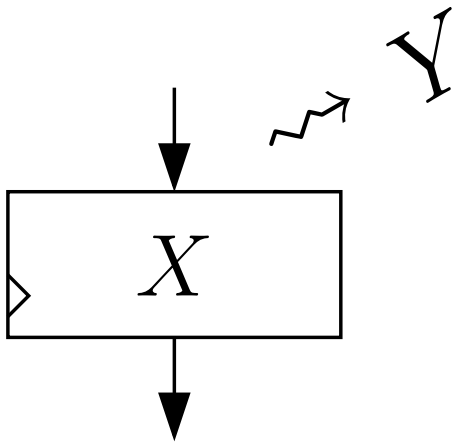
Presentation Outline

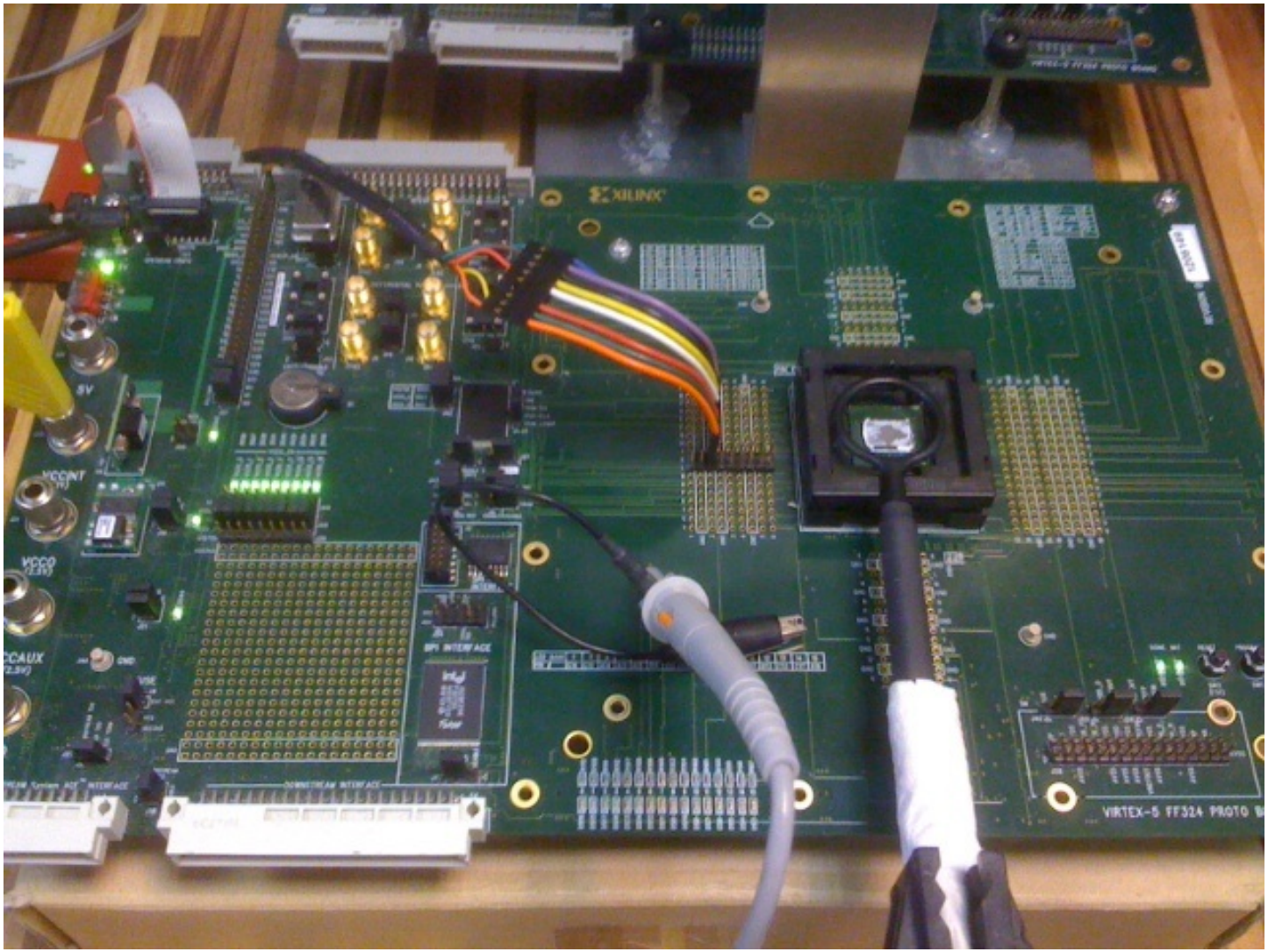
Masking (1/3)

Fact:

manipulating a variable leaks.

$X \rightsquigarrow Y$



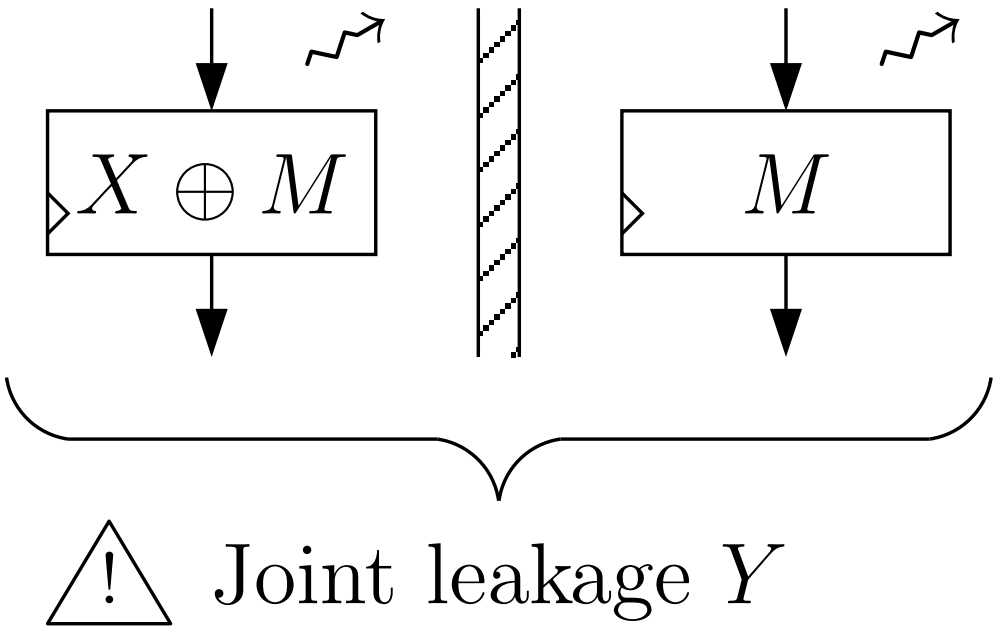

XILINX
VIRTEX-5 FF324 PROTO

Masking (2/3)

Fact:

manipulating a variable leaks.

$X \rightsquigarrow Y$



Masking (3/3)

Two drawbacks in this approach (namely: $Y = \mathrm{HW}(X \oplus M) + N$):

1. Existance of second-order attacks
2. Cost of masking non-linear functions ($n \times n \quad \rightarrow \quad 2n \times 2n$)

Unless...

1. Secure S-box computation ..... [RP10]
2. H-tables ..... [Cor13]

... long to execute (but a nice software protection)

Objective: minimize the attack **degree** (not the order!)

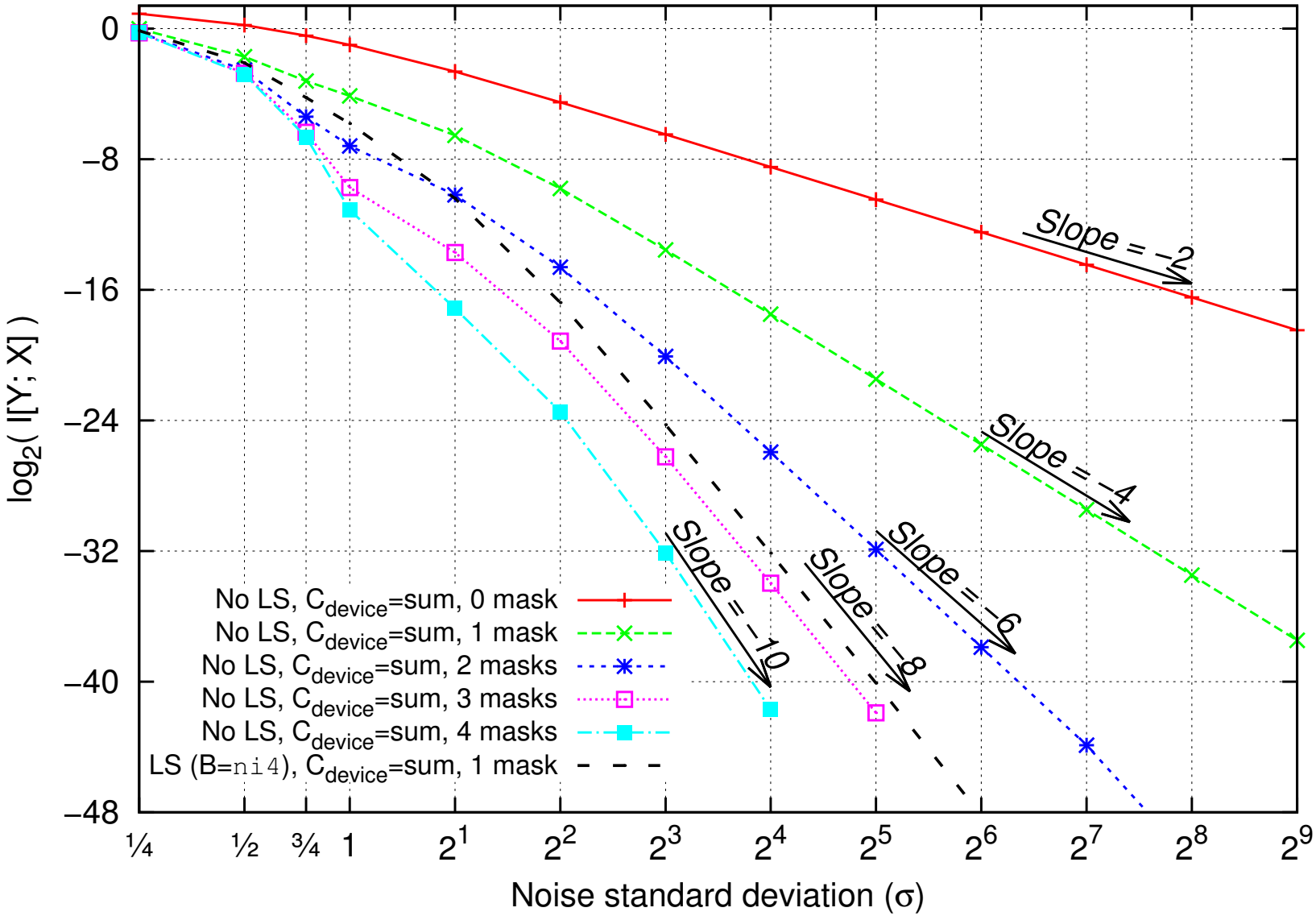
- Maximize d such as $\mathrm{Var}[\mathbb{E}[Y^d \mid X]] = 0$.
- We define Higher-order Correlation Immunity (HCI, [CDG+14]) as:

$$\mathrm{HCI} = \min\{d \in \mathbb{N}^\star; \mathrm{Var}[\mathbb{E}[Y^d \mid X]] > 0\} \ .$$

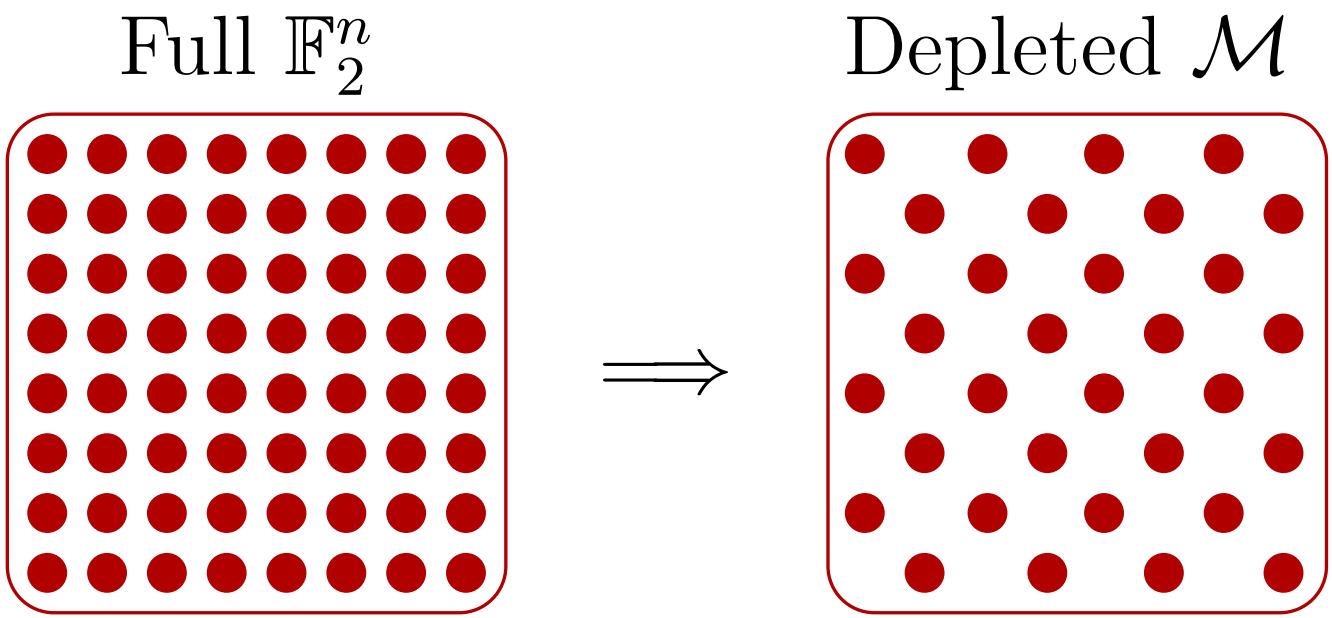
- Theorem 1 in [CDG+14]:

Theorem

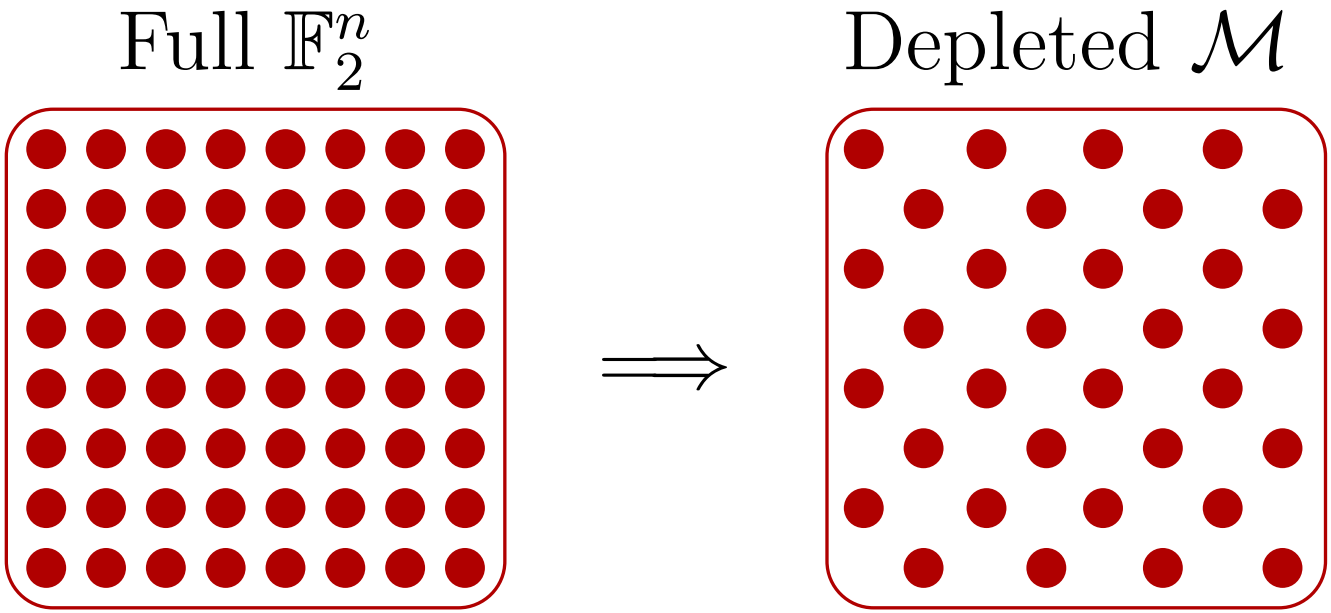
Let σ denote the standard deviation of the noise N, the mutual information $\mathrm{I}[X; Y]$ tends towards $\mathcal{O}\left(\sigma^{-2\times \mathrm{HCI}}\right)$ when σ tends towards infinity.


0
−8
−16
−24
−32
−40
−48
log2(I[Y; X])
¼
½
¾
1
2^1
2^2
2^3
2^4
2^5
2^6
2^7
2^8
2^9
Noise standard deviation (σ)
Slope = −2
Slope = −4
Slope = −6
Slope = −8
Slope = −10
No LS, Cdevice=sum, 0 mask
No LS, Cdevice=sum, 1 mask
No LS, Cdevice=sum, 2 masks
No LS, Cdevice=sum, 3 masks
No LS, Cdevice=sum, 4 masks
LS (B=ni4), Cdevice=sum, 1 mask

LEMS . . . . . Principle



LEMS . . . . . Principle



E.g., $\mathcal{M} = \{\texttt{0x00}, \texttt{0xff}\} \implies \mathsf{HCI} = 2$

Proof of the two-masks protection [BDGN13]

$$
\begin{aligned}
& \mathrm{Var}[\mathbb{E}[Y^1 \mid X]] = 0 \\
\iff & \forall x \in \mathbb{F}_2^n,\ \mathbb{E}[Y \mid X = x] = \mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ \mathbb{E}[\mathrm{HW}(X \oplus M) \mid X = x] = \mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ \mathbb{E}[\mathrm{HW}(x \oplus M)] = \mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ \sum_{m \in \{\mathtt{0x00}, \mathtt{0xff}\}} \frac{1}{2}\mathrm{HW}(x \oplus m) = \mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ \mathrm{HW}(x \oplus \mathtt{0x00}) + \mathrm{HW}(x \oplus \mathtt{0xff}) = 2\mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ \mathrm{HW}(x) + \mathrm{HW}(\neg x) = 2\mathbb{E}[Y] \\
\iff & \forall x \in \mathbb{F}_2^n,\ n = 2\mathbb{E}[Y]
\end{aligned}
$$

It is easy to prove that it applies also to $Y = \sum_{i=1}^{n} \alpha_i x_i$, $\alpha_i \in \mathbb{R}$.

RSM: Rotating Sboxes Masking . . . . . Implementation

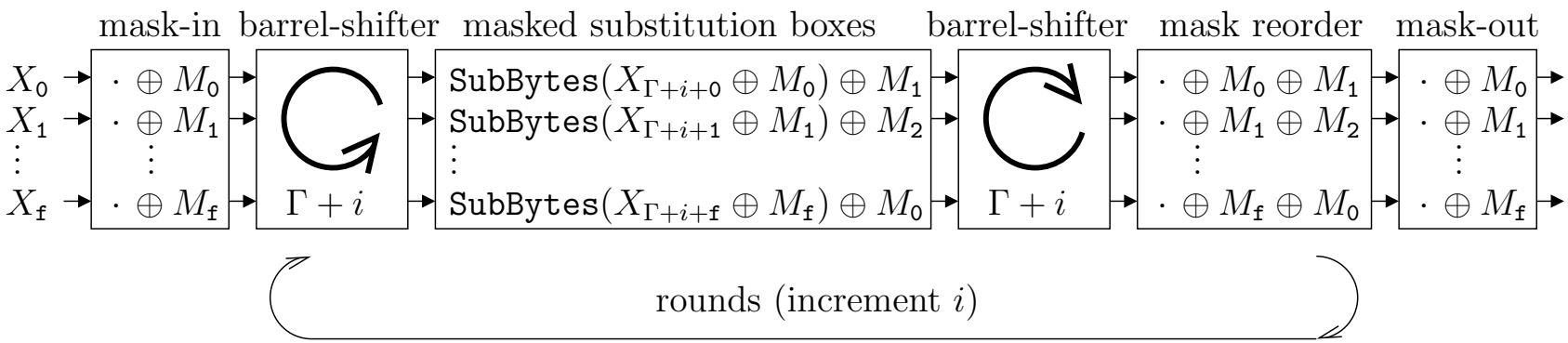
Leakage Model

$Y = \mathrm{HW}(M \oplus X)$, where $M \sim \mathcal{U}(\mathcal{M})$, with $\mathcal{M} \subseteq \mathbb{F}_2^n$.

Solution

The scheme is dth-degree secure if $1_{\mathcal{M}} : \mathbb{F}_2^n \to \mathbb{F}_2$ is dth-order correlation immune.

For AES: 16 identical S-boxes = SubBytes.



Equivalent properties of a code C and its indicator function f [MS77].

Code $C \subseteq \mathbb{F}_2^n$	**Indicator** $f : \mathbb{F}_2^n \to \mathbb{F}$ **of** C
C has size $w = \mathrm{Card}[C]$	f has weight $w = \widehat{f}(0)$
C has dual distance $d_C^\perp$	f is $(d_C^\perp - 1)$-CI and not $d_C^\perp$-CI
$B_d^\perp = \sum_{\substack{z \in \mathbb{F}_2^n, \text{ s.t.} \\ \mathrm{HW}(z)=d}} \left(\frac{1}{\mathrm{Card}[C]} \sum_{x \in C} (-1)^{x \cdot z} \right)^2$	$B_d^\perp = \sum_{\substack{z \in \mathbb{F}_2^n, \text{ s.t.} \\ \mathrm{HW}(z)=d}} \left(\widehat{f}(z) / \widehat{f}(0) \right)^2$

"*Implementation attacks*" side		"*Countermeasures*" side
Resistance at degree d to high-order correlation attacks		The indicator of the masks $\mathcal{M} = \text{supp}(M)$ is a dth-order correlation-immune Boolean function
$\forall i \in [\![1, d]\!]$, $\text{Var}[\mathbb{E}[w_H(X \oplus M)^i \mid X]] = 0$	$\Longleftrightarrow$	$\forall x \in \mathbb{F}_2^n, 1 \leq w_H(x) \leq d$, $\sum_{m \in \mathcal{M}} (-1)^{x \oplus m} = 0$

$n = 8$, $|\mathcal{C}| = 2^4$: Boolean linear code $[8, 4, 4]$

Dual distance $= 4$:

0x00	0	0	0	0	0	0	0	0
0x0f	0	0	0	0	1	1	1	1
0x36	0	0	1	1	0	1	1	0
0x39	0	0	1	1	1	0	0	1
0x53	0	1	0	1	0	0	1	1
0x5c	0	1	0	1	1	1	0	0
0x65	0	1	1	0	0	1	0	1
0x6a	0	1	1	0	1	0	1	0
0x95	1	0	0	1	0	1	0	1
0x9a	1	0	0	1	1	0	1	0
0xa3	1	0	1	0	0	0	1	1
0xac	1	0	1	0	1	1	0	0
0xc6	1	1	0	0	0	1	1	0
0xc9	1	1	0	0	1	0	0	1
0xf0	1	1	1	1	0	0	0	0
0xff	1	1	1	1	1	1	1	1

Properties of the code [CG14b]

$$G = \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}\right) .$$

generates a *self-dual* code and *BKLC*.

Proposition

There are only three different distributions of $\mathsf{HW}(y \oplus M)$*, when* $M \sim \mathcal{U}(C)$ *when* $y \in \mathbb{F}_2^8$*. Namely, the set of probabilities* $(p(\mathsf{HW}(y \oplus M) = \ell))_{\ell \in [\![0,8]\!]}$ *are equal to:*

1. $\left(\frac{1}{16}, 0, 0, 0, \frac{14}{16}, 0, 0, 0, \frac{1}{16}\right)$ *if* $y \in C$*;*
2. $\left(0, \frac{1}{16}, 0, \frac{7}{16}, 0, \frac{7}{16}, 0, \frac{1}{16}, 0\right)$ *if there exists a codeword of Hamming weight* 1 *in* $y \oplus C$*;*
3. $\left(0, 0, \frac{4}{16}, 0, \frac{8}{16}, 0, \frac{4}{16}, 0, 0\right)$ *if there exists a codeword of Hamming weight* 2 *in* $y \oplus C$*.*

$$\mathrm{Var}[\mathbb{E}[\mathscr{L}^{d+1}|Z]] = \frac{1}{2^{2n}} \sum_{\substack{z\in\mathbb{F}_2^n, \text{ s.t.} \\ \mathrm{HW}(z)=d+1}} \left(\frac{\widehat{f}(z)}{\widehat{f}(0)} \cdot \widehat{\mathcal{L}^{d+1}}(z) \right)^2 . \qquad (3)$$

For a Hamming weight leakage:

$$\forall d < d_C^\perp, \mathrm{Var}[\mathbb{E}[\mathscr{L}^d|Z]] = 0 \quad \text{and} \quad \mathrm{Var}[\mathbb{E}[\mathscr{L}^{d_C^\perp}|Z]] = B_{d_C^\perp}^\perp \left(\frac{d_C^\perp !}{2^{d_C^\perp}} \right)^2 .$$

$$\mathsf{I}[\mathscr{L}+N;Z] = \frac{1}{\ln 2}\sum_{d=0}^{+\infty}\frac{1}{2\,d!}\sum_{z\in\mathbb{F}_2^n} p(Z=z)\frac{\left(k_d(\mathscr{L}\mid Z=z)-k_d(\mathscr{L})\right)^2}{\left(\mathrm{Var}[\mathscr{L}]+\sigma^2\right)^d}$$

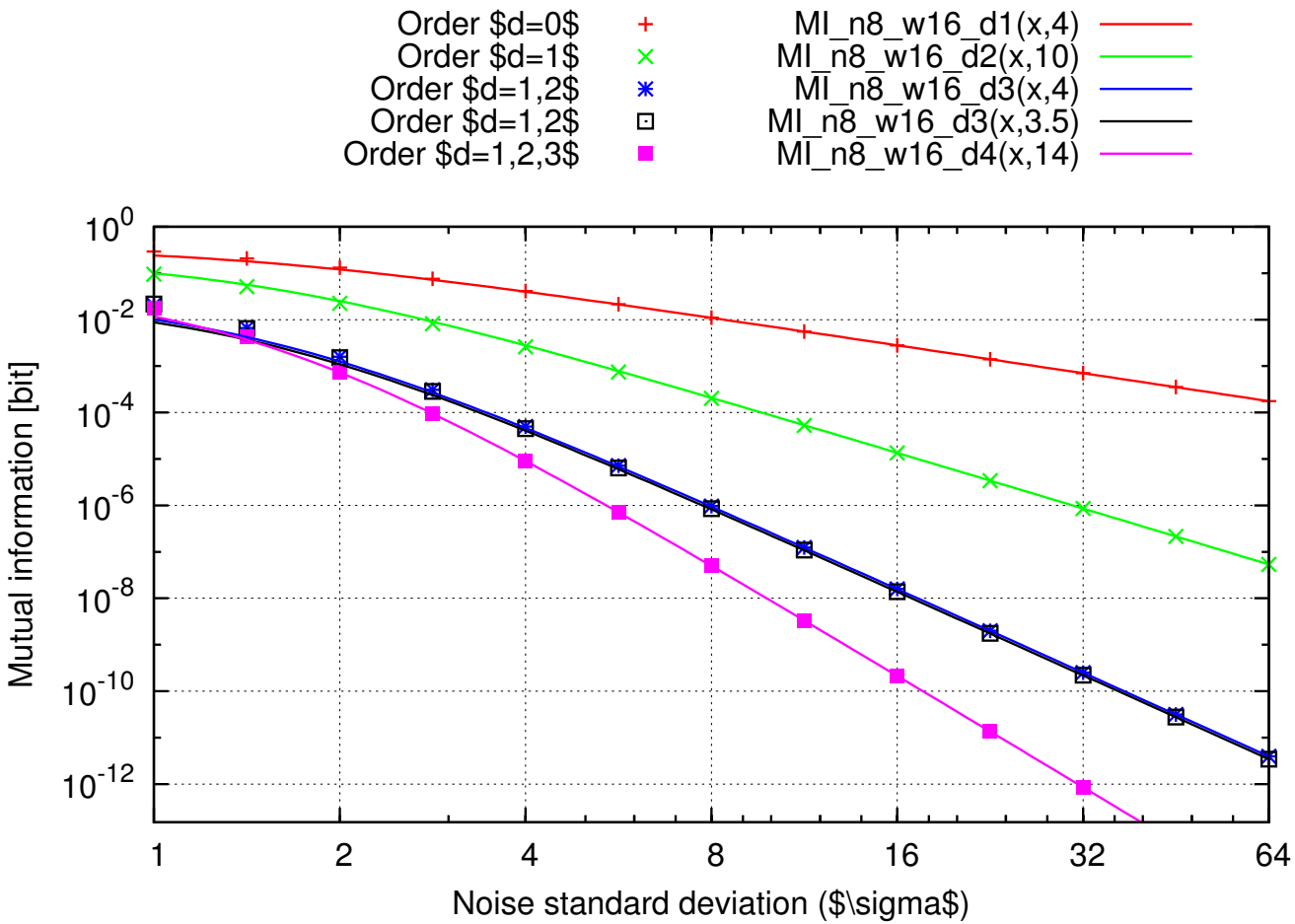
$$= \frac{1}{\ln 2}\sum_{d=0}^{+\infty}\frac{1}{2\,d!}\frac{\mathrm{Var}[\mathbb{E}[k_d(\mathscr{L}\mid Z)]]}{\left(\mathrm{Var}[\mathscr{L}]+\sigma^2\right)^d}\quad , \tag{4}$$

$$= \frac{d_C^{\perp}!B_{d_C^{\perp}}^{\perp}}{2\ln 2\cdot 2^{2d_C^{\perp}}}\times\frac{1}{\sigma^{2d_C^{\perp}}}+\mathcal{O}\left(\frac{1}{\sigma^{2(d_C^{\perp}+1)}}\right)\quad \text{when } \sigma\to\infty\ . \tag{5}$$

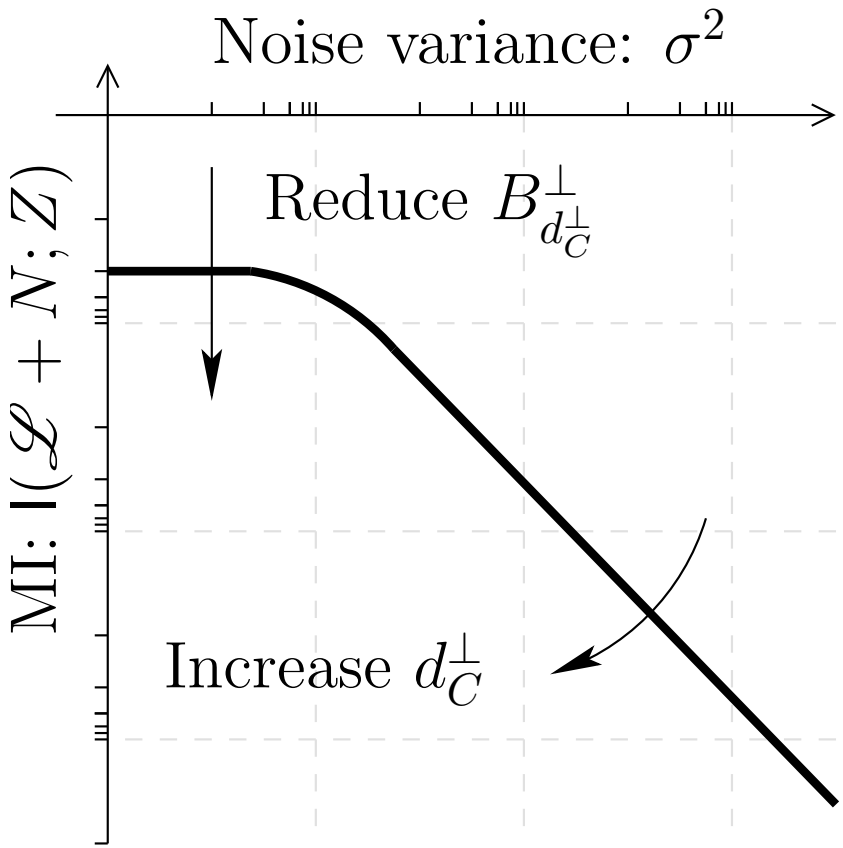
where k_d are order d cumulants [LB10].

Coefficients of the distance enumerator polynomial for some codes ($B^{\perp}_{d^{\perp}_C}$ in **bold**).

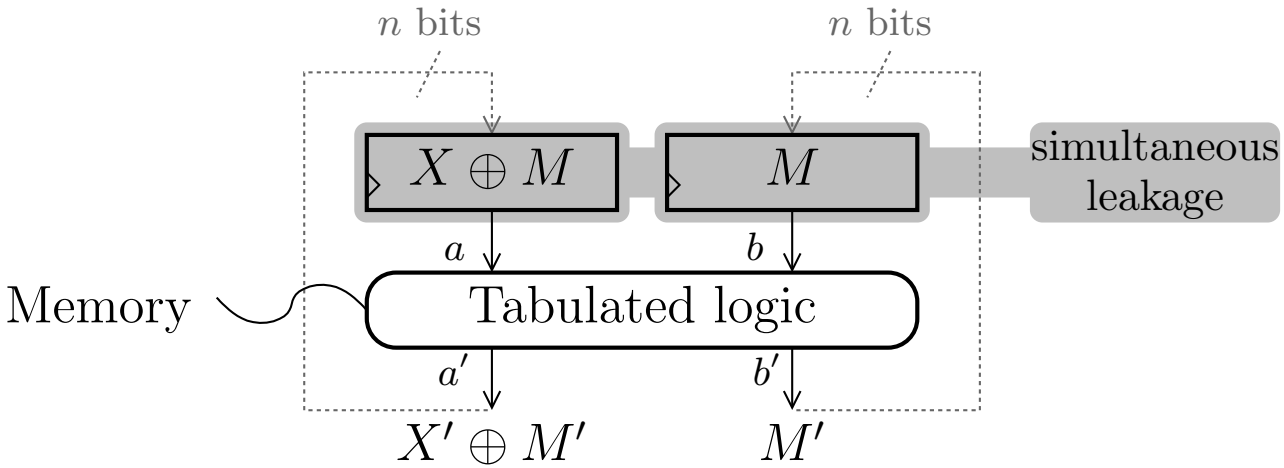
Code #	$B^{\perp}_0$	$B^{\perp}_1$	$B^{\perp}_2$	$B^{\perp}_3$	$B^{\perp}_4$	$B^{\perp}_5$	$B^{\perp}_6$	$B^{\perp}_7$	$B^{\perp}_8$
1	1	**8**	28	56	70	56	28	8	1
2	1	0	**28**	0	70	0	28	0	1
3	1	0	0	**3.5**	7	3.5	0	0	1
4	1	0	0	**4**	5	4	2	0	0
5	1	0	0	0	**14**	0	0	0	1



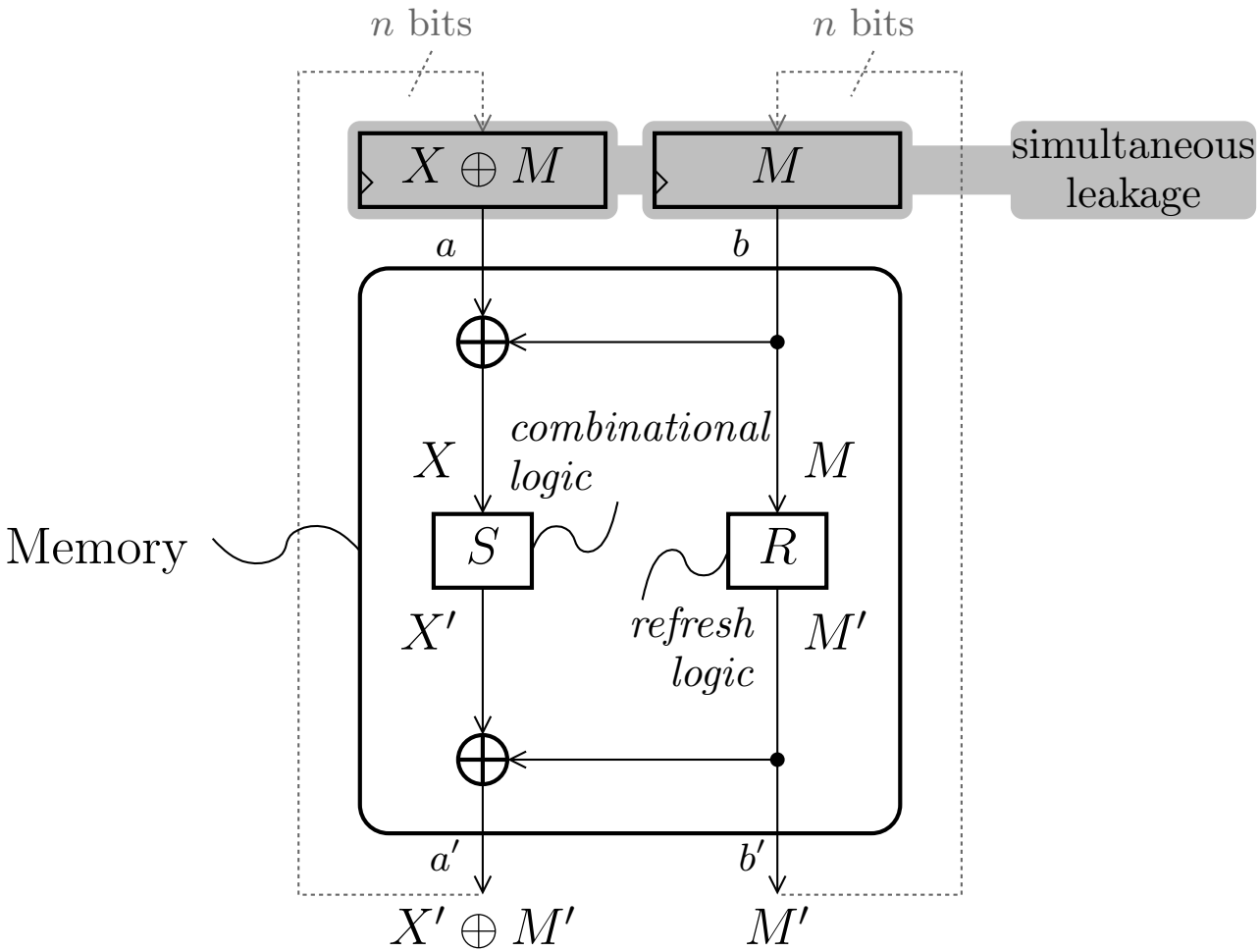
Approximation of $\mathrm{I}[\mathscr{L} + N; Z]$ by $\left(d_C^{\perp} ! B_{d_C^{\perp}}^{\perp}\right) \Big/ \left(2 \ln 2 \cdot 2^{2d_C^{\perp}} \sigma^{2d_C^{\perp}}\right)$.



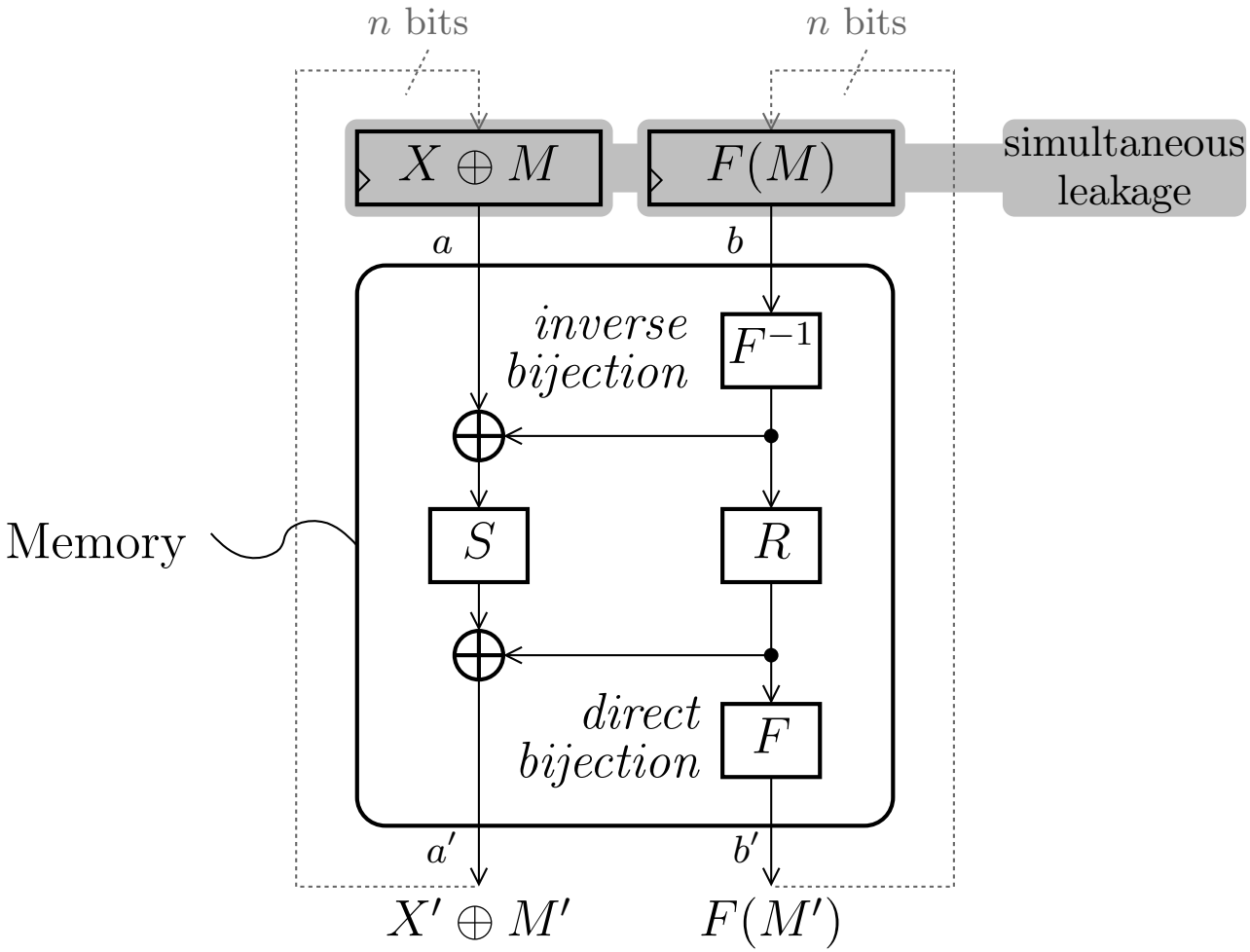
Two concomitant objectives to reduce the mutual information.



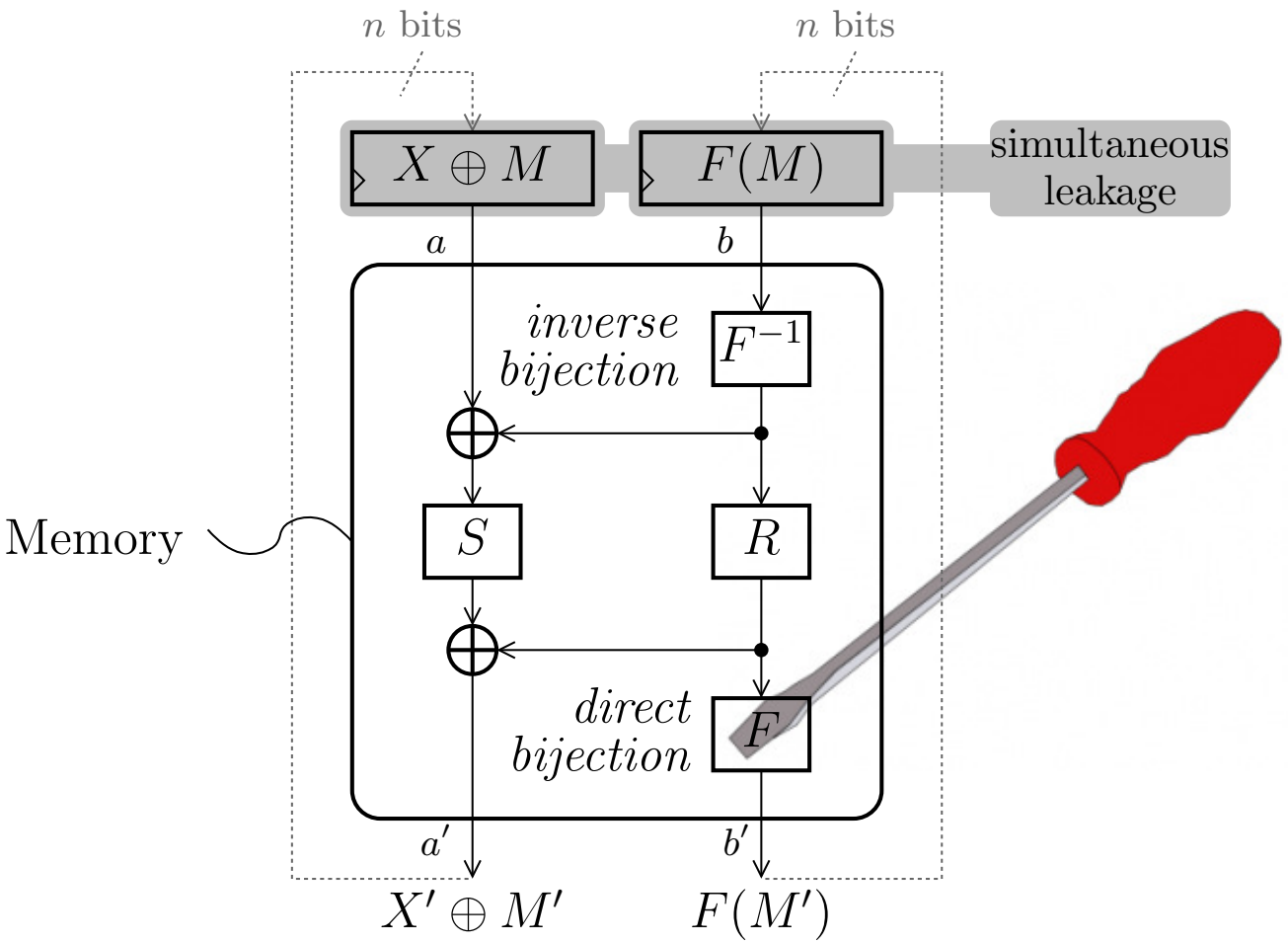
Principle of masking without throughput loss.



Detail of the computation.



Principle of leakage squeezing, within throughput loss.


n bits
n bits
$X \oplus M$
$F(M)$
simultaneous leakage
a
b
inverse bijection
F^{-1}
Memory
S
R
direct bijection
F
a'
b'
$X' \oplus M'$
$F(M')$

Objective

Maximize d such as $\mathrm{Var}[\mathbb{E}[(\mathrm{HW}(X \oplus M, F(M)))^d \mid X]] = 0$.

Solution

Resistance order d: $1 \leq d \leq n-1$, where $d+1$ is the maximal dual distance of codes $(2n, 2^n, \delta)$ with complementary information sets.

The code is the support of the indicator of the graph of F.

Examples

- Optimal code: NR $(16, 2^8, 6)$ for $n = 8$. See [CGKS12] $(\forall n)$
- High-order CIS codes [CDGM12, CFG$^+$].

Presentation Outline

Theorem (rank-nullity)

$\dim(\mathcal{C}) + \dim(\mathcal{C}^\perp) = \dim(\mathbb{F}_2^n) = n$, *where* $\dim(\cdot)$ *is the dimension of the vector space.*

As a direct consequence of Theorem 6, we have $\dim(\mathcal{C}^\perp) = n - k$.

Remark

However, $\mathcal{C}$ *and* $\mathcal{C}^\perp$ *are not necessarily supplementary, i.e., we do not have* $\mathcal{C} \cap \mathcal{C}^\perp = \{0\}$*. For instance, if* $\mathcal{C}$ *is autodual, then* $\mathcal{C} = \mathcal{C}^\perp$*.*

Indeed, unlike in Euclidean spaces, the scalar product does not define a norm.

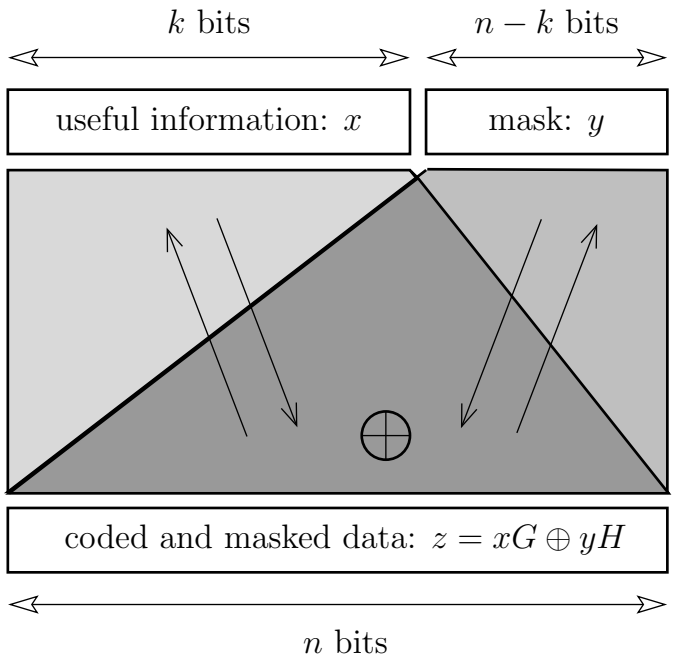
Proposition (Condition for $\mathbb{F}_2^n = \mathcal{C} \oplus \mathcal{C}^\perp$)

Without loss of generality (a permutation of coordinates might be necessary), we can assume that the generating matrix of $\mathcal{C}$ is systematic, and thus takes the form $[I_k \| M]$, where I_k is the $k \times k$ identity matrix. The supplementary $\mathcal{D}$ of $\mathcal{C}$ is equal to $\mathcal{C}^\perp$ if and only if (iff) the matrix $I_k \oplus MM^\mathsf{T}$ is invertible.

When $\mathcal{D} = \mathcal{C}^{\perp}$, there is an orthogonal projection. Indeed, we thus have $GH^{\mathsf{T}} = 0$ (the all-zero $k \times (n-k)$ matrix). In this case, H is the *parity matrix* of code $\mathcal{C}$. So, in Eq. (2), x and y can be recovered from z, as follows:

$$x = zG^{\mathsf{T}}(GG^{\mathsf{T}})^{-1} \ , \tag{6}$$

$$y = zH^{\mathsf{T}}(HH^{\mathsf{T}})^{-1} \ . \tag{7}$$



Computing in ODSM

$$L' = G^{\mathsf{T}} \left(G \cdot G^{\mathsf{T}} \right)^{-1} LG \oplus H^{\mathsf{T}} \left(H \cdot H^{\mathsf{T}} \right)^{-1} H \ . \tag{8}$$

$$\forall z \in \mathbb{F}_2^n, \quad S'(z) = S(zG^{\mathsf{T}}(GG^{\mathsf{T}})^{-1})G \oplus zH^{\mathsf{T}}(HH^{\mathsf{T}})^{-1}H \ , \tag{9}$$

SCA resistance

ODSM can be attacked by monovariate high-order SCA only at order $j \geq d_{\mathcal{C}}$.

DFA resistance

$$P_{\mathcal{D}}(z) \stackrel{?}{=} yH \ . \tag{10}$$

We consider a perturbation as the addition to the state z of a random error ε ($z \leftarrow z \oplus \varepsilon$). Like z (recall $z = xG \oplus yH$, see Eq. (2)), the fault can be uniquely written as:

$$\varepsilon = eG \oplus fH \ , \text{ where } e \in \mathbb{F}_2^k \text{ and } f \in \mathbb{F}_2^{n-k} \ . \tag{11}$$

The fault is undetected if
$P_{\mathcal{D}}(z \oplus \varepsilon) = (y \oplus f)H = yH \Leftrightarrow f = 0 \Leftrightarrow \varepsilon \in \mathcal{C}$ (**difficult!**)

Notations

- x is encoded: $\Psi(x) = xG \in \mathbb{F}_2^n$
- $\Psi(x)$ is manipulated masked by some $d \in \mathcal{D}$
- Indicator $1_{\mathcal{D}}$ of $\mathcal{D}$ is noted $f : \mathbb{F}_2^n \to \mathbb{F}_2$:

$$\forall d \in \mathbb{F}_2^n, \quad f(d) = 1 \iff d \in \mathcal{D} \ .$$

Said differently, $f(d) = 1 \iff \exists y \in \mathbb{F}_2^{n-k}$ s.t. $yH = d$.

Statistics

- Random variables: D
- Realizations: d
- Support: $\mathcal{D}$

Leakage model / attacker model

- We model the attacker as a pseudo-Boolean function $\Phi : \mathbb{F}_2^n \to \mathbb{R}$ of a given numerical degree j in the bits of Z.
- For example, Φ can be the power j of the Hamming weight (as in zero-offset attacks).
- The leakage model can be, in general, any affine function of the bits of Z. This simply means that there is no "glitch" nor "cross-couping". This case is usual for software platforms, and in hardware when using memories.

Proposition (jth-order security condition on the masks coding)

Let $\Phi : \mathbb{F}_2^n \to \mathbb{R}$ a leakage function of numerical degree j, an arbitrary $\Psi : \mathbb{F}_2^k \to \mathbb{F}_2^n$ and a mask D uniformly distributed in a code $\mathcal{D}$, with f the indicator of $\mathcal{D} \subset \mathbb{F}_2^n$. Then the leakage $\Phi(\Psi(X) \oplus D)$ resists a monovariate attack if $\mathcal{D}$ is a code of dual distance $j+1$.

In Proposition 4, the condition of jth-order security is: for all Φ of numerical degree smaller than or equal to j, $\mathbb{E}[\Phi(\Psi(X) \oplus D)|X = x]$ does not depend on $x \in \mathbb{F}_2^k$. This is rewritten as the condition:

$$\mathrm{Var}[\mathbb{E}[\Phi(\Psi(X) \oplus D)|X]] = 0 \ . \tag{12}$$

Indeed, in this case, any correlation attack fails: indeed, there is no linear dependency between the leakage $\Phi(\Psi(X) \oplus D)$ and the sensitive variable X.

Now, the expectation $\mathbb{E}[\Phi(\Psi(X) \oplus D)|X = x]$ is taken on the mask D random variable only, because $\Psi(X)$ depends only on X. So we have:

$$\begin{aligned}\mathbb{E}[\Phi(\Psi(X) \oplus D)|X = x] &= \sum_{d \in \mathcal{D}} \frac{1}{\mathrm{Card}[\mathcal{D}]} \Phi(\Psi(x) \oplus d) \\ &= 2^{-(n-k)} \sum_{d \in \mathbb{F}_2^n} f(d)\Phi(\Psi(x) \oplus d) \\ &= 2^{-(n-k)} \left(f \otimes \Phi\right)\left(\Psi(x)\right) \ .\end{aligned}$$

So, the countermeasure is jth-order secure if and only if $\left(f \otimes \Phi\right)\left(\Psi(x)\right)$ does not depend on x. Therefore, a sufficient condition for resistance against jth-order attacks is that $f \otimes \Phi(z)$ does not depend on $z \in \mathbb{F}_2^n$ (irrespective of function Ψ).

Let g a pseudo-Boolean function $\mathbb{F}_2^n \to \mathbb{R}$. We call $\hat{g}$ the Fourier transform of g, i.e., $\hat{g}(z) = \sum_a g(a)(-1)^{a \cdot z}$. We have:
(g is constant) $\iff \forall z \neq 0,\ \hat{g}(z) = 0 \iff \hat{g} \propto \delta$, the Kronecker symbol.
Let us apply this result to $g = f \otimes \Phi$. The Fourier transform turns a *convolution product* into a *product*, i.e., $\widehat{f \otimes \Phi}(z) = \hat{f}(z)\hat{\Phi}(z)$. To prove that:

$$\hat{f}\,\hat{\Phi} = 0 \ , \tag{13}$$

Lemma

Lemma

Let P be a pseudo-Boolean function $P : \mathbb{F}_2^n \to \mathbb{R}$ of numerical degree $d^\circ(P)$ [Car10, CG99]. Then, $\forall z \in \mathbb{F}_2^n$,
$\mathrm{HW}(z) > d^\circ(P) \Longrightarrow \widehat{P}(z) = 0.$

Proof.

(Proof of Proposition 4) So, to prove that $\widehat{f}\,\widehat{\Phi} = 0$, we start by applying Lemma 7. As Φ is of numerical degree j, $\widehat{\Phi}(z) = 0$ for $\mathrm{HW}(z) > j$. So, the masking is jth-order secure if $\forall z \in \mathbb{F}_2^n$, $0 < \mathrm{HW}(z) \leq j$, $\widehat{f}(z) = 0$. By definition, this means that f is jth-order correlation-immune (j-CI in brief). This is equivalent to saying the $\mathcal{D}$ is of dual distance $d_{\mathcal{D}}^{\perp} = j + 1$.
Irrespective of the way the sensitive variable $X \in \mathbb{F}_2^k$ is mapped (by function Ψ) onto $\mathbb{F}_2^n$, a sufficient condition for security against zero-offset attacks [WW04] of orders $1, 2, \cdots, j$ is that the mask D be distributed uniformly in $\mathcal{D}$, a code of dual distance $j + 1$. Said differently, the lowest order j of a successful zero-offset attack is equal to the dual distance of $\mathcal{D}$.
As $\mathcal{D} = \mathcal{C}^{\perp}$, we have that $d_{\mathcal{D}}^{\perp} = d_{\mathcal{C}}$ (see Proposition 1). □

$$G = \left(\begin{array}{cccccccc|cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1
\end{array}\right),$$

$$H = \left(\begin{array}{cccccccc|cccccccc}
1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right).$$

$$G^{\mathsf{T}}(GG^{\mathsf{T}})^{-1} =$$

$$\begin{pmatrix}
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}$$

$$H^{\mathsf{T}}(HH^{\mathsf{T}})^{-1} =$$

$$\begin{pmatrix}
0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 1
\end{pmatrix}$$

xtime: multiplication by X in $\mathbb{F}_2^8$

Generated from this $k \times k$ (i.e., 8×8) matrix L:

$$L = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

$$L' = \left(\begin{array}{cccccccc|cccccccc}
1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
\hline
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0
\end{array}\right).$$

Conclusions

- Codes can help increase the security level:
  - Against side-channel attacks, and
  - Against fault injection attacks
- Protection against multivariate attacks: is a perspective

Thank you!

- $x \in \mathbb{F}_2^k$ the clear data,
- $y = (y_1, y_2, \ldots, y_d) \in (\mathbb{F}_2^k)^d$ are the masks, and the protected data is:
- $z = (x + \sum_{i=1}^{d} y_i, y_2, \ldots, y_d)$.

So we have $n = tk$ $(t = d + 1)$, and $z = xG + yH$, where

$$G = \begin{pmatrix} I & 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$H = \begin{pmatrix} I & I & 0 & \cdots & 0 \\ I & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & 0 & 0 & \cdots & I \end{pmatrix}.$$

Notice that $GH^{\mathsf{T}} \neq 0$, thus the codes generated by G and H are not supplementary dual.

Security equivalence

We have proved with OSDM that the representation $z = xG + yH$ protects against attacks of order d if $d_D^\perp$ is of dual distance at least t, where D is the linear code of generator matrix H. Notice that $d_D^\perp \neq d_C$ since $D \neq C^\perp$ (unlike ODSM).

Remark

The dual distance of the linear code of generator matrix H is t. Indeed, the dual code of the code generated by H is generated by:

$$H^\perp = \begin{pmatrix} I & I & I & \cdots & I \end{pmatrix}, \tag{14}$$

which has minimal distance t (it is the t-wise repetition code).

Remark

This code is not optimal, unless $k = 1$.

Inverse coding

- We have $z = \begin{pmatrix} x & y \end{pmatrix} M$, where the matrix M is equal to $M = \begin{pmatrix} G \\ H \end{pmatrix}$.
- Therefore, $\begin{pmatrix} x & y \end{pmatrix} = zM^{-1}$, where $M^{-1} = M$ (the coding is *involutive*).

$$M^{-1} = \left(\begin{array}{c|cccc} I & 0 & 0 & \cdots & 0 \\ I & I & 0 & \cdots & 0 \\ I & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & 0 & 0 & \cdots & I \end{array}\right) = \begin{pmatrix} J & K \end{pmatrix}$$

$$\Longrightarrow \; x = Jz, y = Kz$$

Unification for the refresh

In all the schemes, the mask refresh is: $y' \in_{\mathcal{R}} \mathbb{F}_2^{n-k}$, $z \leftarrow z + y'H$.

C14

Remark (Steps of C14 for $d = 1$)

1. $\forall u \in \mathbb{F}_2^k,\ T(u) = S(u)G,$
2. $\forall u \in \mathbb{F}_2^k,\ T(u) = S(u + x_1)G + y_1 H,$
3. $\forall u \in \mathbb{F}_2^k,\ T(u) = S(u + x_1 + x_2)G + (y_1 + y_2)H,$
4. *return* $T(0)$

Somehow, if we collapse the iterations, C14 consists in computing: $S(zJ)G + (y_1 + y_2)KH$.

Countermeasures against Physical Attacks using Error-Correcting Codes

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